



Influence of acoustic pressure amplifier dimensions on the performance of a standing-wave thermoacoustic system

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ABSTRACT

A standing-wave thermoacoustic engine, employing an acoustic pressure amplifier (APA), is simulated with linear thermoacoustics to study the influence of APA's dimensions on performance of the thermoacoustic system. Variations of operating parameters, including pressure ratio, acoustic power, hot end temperature of stack etc., versus length and diameter of APA are presented and discussed based on an analysis of pressure and velocity distribution in APA. Simulation results indicate that a largest amplification effect of both pressure ratio and acoustic power output is achieved at a critical length for the occurrence of pressure node and velocity antinode in APA, close to but less than one fourth of the wavelength. The distribution characteristics of pressure and velocity in APA are similar to a standing-wave acoustic field, which is the reason for the amplification effect. From the viewpoint of energy, the amplification effect results from the changed distribution of acoustic energy and acoustic power loss in the thermoacoustic system by APA. Experiments have been carried out to validate the simulation, and experimental data are presented.

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1. Introduction

A thermoacoustic engine is a novel prime mover without any moving component, whose potential of high reliability and long life attracts academic and industrial interests. In recent years, much effort has been focused on the performance enhancement of thermoacoustic engines [1–13]. Dai and Luo et al. introduced an acoustic pressure amplifier (APA), a long circular tube (see Fig. 1), to a thermoacoustic engine in 2005 [8]. This configuration was expected to elevate pressure ratio (maximum pressure divided by minimum pressure of pressure oscillation), leading to a higher pressure ratio at APA's end connecting to the load compared with that at its other end connecting to the engine, which is referred to as acoustic pressure amplification effect. The elevated pressure ratio is of great importance for improving the performance of thermoacoustic engine driving a load, e.g., a pulse tube cooler, a thermoacoustic cooler or an electrical generator. An increase of pressure ratio from 1.15 to 1.32 by an APA of 2.8 m in length and 8 mm in diameter was achieved in a traveling-wave thermoacoustic engine [8], which then led to a cooling temperature of a single-stage pulse tube cooler down to 65.7 K. The introduction of an APA of 3.4 m in length and 8 mm in diameter between a standing-wave thermoacoustic engine and a pulse tube cooler increased the pressure ratio from 1.129 to 1.152 and lowered the cooling

temperature from 88.6 K to 79.7 K [10]. So far, single-stage and two-stage pulse tube coolers driven by traveling-wave thermoacoustic engine with suitable APA have reached 34.1 K and 18.7 K [11], respectively. And a cooling temperature of 56.4 K has been achieved from a single-stage pulse tube cooler driven by a standing-wave thermoacoustic engine [13]. However, experiments also indicate that the amplification effect is sensitive to the dimensions of APA, and unsuitable dimensions may cause the failure in pressure ratio amplification [10].

As for the simulation, Dai and Luo et al. calculated the acoustic pressure amplification effect of an APA (for driving a pulse tube cooler) with linear thermoacoustics [8]. Bao et al. focused on the computation of amplified pressure ratio by an APA without any load [10]. However, the impact of APA on the operation of thermoacoustic engine, including pressure oscillation in the engine and hot end temperature of stack etc. are seldom discussed. Besides, the mechanism of APA's amplification effect also needs further analysis.

In order to study the influence of APA's dimensions on the performance of thermoacoustic system and to supply useful information for optimal design, a standing-wave thermoacoustic engine driving a resistance-and-compliance (RC) load through an APA was simulated with linear thermoacoustics [14]. Based on analysis of distribution characteristics of pressure and velocity, variation of operating parameters versus APA's length and diameter has been discussed. The mechanism of amplification effect of pressure ratio and acoustic power output by APA is analyzed. Some experiments

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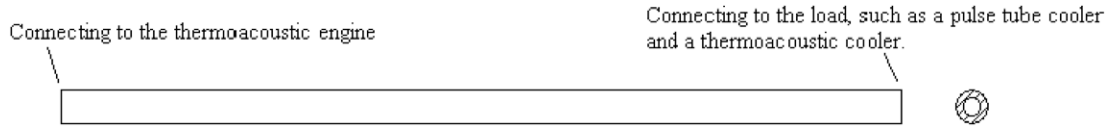


Fig. 1. Schematic of acoustic pressure amplifier (APA).

have also been carried out to validate the simulation, and experimental data are presented.

2. Simulation of thermoacoustic engine driving an RC load through an APA

The standing-wave thermoacoustic engine driving an RC load through an APA is schematically shown in Fig. 2. The engine employs a symmetrical configuration. Main dimensions are listed in Table 1. The APA is a long circular tube. In the simulation, impedance of the RC load is fixed $1 \times 10^9 - i \times 1 \times 10^8 \text{ Pa} \cdot \text{s/m}^3$ in order to focus emphasis on the impact of APA's dimensions, i.e., length and diameter.

According to linear thermoacoustics [14], the momentum, continuity and energy equations for a short channel are as follows:

$$\frac{dp_1}{dx} = -(i\omega l + r_v)U_1 \quad (1)$$

$$\frac{dU_1}{dx} = -\left(i\omega c + \frac{1}{r_k}\right)p_1 + eU_1 \quad (2)$$

$$\frac{dT_m}{dx} = \frac{\dot{H}_2 - \frac{1}{2}\text{Re}\left[p_1 \tilde{U}_1 \left(1 - \frac{f_k - \tilde{f}_v}{(1+\xi)(1+Pr)(1-\tilde{f}_v)}\right)\right]}{\frac{\rho_m c_p |U_1|^2}{2A\omega(1-Pr)|1-\tilde{f}_v|^2} \text{Im}\left(\tilde{f}_v + \frac{(f_k - \tilde{f}_v)(1+\xi\tilde{f}_v/f_k)}{(1+\xi)(1+Pr)}\right) - (Ak + A_{\text{solid}}k_{\text{solid}})} \quad (3)$$

$$l = \frac{\rho_m}{A} \frac{1 - \text{Re}(f_v)}{|1 - \tilde{f}_v|^2} \quad (4)$$

$$c = \frac{A}{\gamma p_m} \left[1 + (\gamma - 1)\text{Re}\left(\frac{f_k}{1 + \xi}\right)\right] \quad (5)$$

$$r_v = \frac{\omega \rho_m}{A} \frac{\text{Im}[-\tilde{f}_v]}{|1 - \tilde{f}_v|^2} \quad (6)$$

$$r_k = \frac{\gamma}{\gamma - 1} \frac{p_m}{\omega A \text{Im}(-\tilde{f}_k/(1 + \xi))} \quad (7)$$

$$e = \frac{f_k - \tilde{f}_v}{(1 - \tilde{f}_v)(1 - Pr)(1 + \xi)} \frac{1}{T_m} \frac{dT_m}{dx} \quad (8)$$

where p_1 and U_1 are pressure amplitude and volumetric velocity amplitude, ω is angular frequency, ρ_m , T_m , p_m , c_p , γ , k and Pr are mean density, temperature, mean working pressure, isobaric specific

Table 1
Dimensions of thermoacoustic engine

	Heater	Stack	Water cooler	Resonant tube	Hot buffer
Diameter (mm)	56	56	56	36	–
Length (mm)	64	285	34	8000	(1.35 L)

heat, specific heat ratio, thermal conductivity and Prandtl number of working fluid, respectively, f_v and f_k are viscous function and thermal function [14], A is flow area of channel, A_{solid} and k_{solid} are cross-section area and thermal conductivity of the solid forming the channel, $\dot{H}_2$ is total power, ξ is a quantity presenting the effect of specific heat and thermal conductivity of the channel solid on the heat transfer between the working fluid and the channel (ξ equals zero for the ideal solid with infinite specific heat and thermal conductivity.) [14], i is imaginary unit, Re , Im and superscript $\sim$ mean the real part, the imaginary part and the conjugation of a complex quantity. l , c , r_v , r_k , and e are inertance, compliance, viscous resistance, thermal-relaxation resistance and proportionality coefficient of a controlled source, which can be calculated with Eqs. (4)–(8).

The expression of acoustic power generation per unit length of stack can be written as

$$\dot{W}_{\text{gen}} = \frac{1}{2} \text{Re}[e \tilde{p}_1 U_1] \quad (9)$$

Acoustic power loss is mainly comprised of viscous loss and thermal-relaxation loss, which can be estimated with Eqs. (10) and (11), respectively.

$$\dot{W}_v = \frac{r_v}{2} |U_1|^2 \quad (10)$$

$$\dot{W}_k = \frac{1}{2r_k} |p_1|^2 \quad (11)$$

The acoustic power delivered to the RC load, i.e., acoustic power loss in the load, can be calculated with Eq. (12).

$$\dot{W}_{\text{load}} = \frac{1}{2} \text{Re}[\tilde{p}_{\text{load}} U_{\text{load}}] \quad (12)$$

where p_{load} and U_{load} are pressure amplitude and velocity amplitude at the inlet of the RC load, respectively.

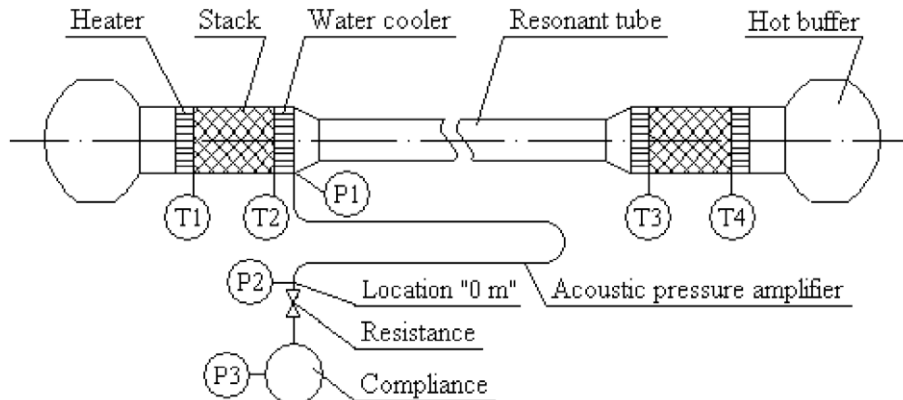


Fig. 2. Schematic of thermoacoustic engine driving an RC (resistance-and-compliance) load through APA. A pulse tube cooler or a thermoacoustic cooler, substituting the RC load, can be connected to the APA at the location "0 m".

Operating parameters of the thermoacoustic system are obtained by solving Eqs. (1)–(3) with finite difference method. In the simulation, each component, except the RC load and the two hot buffers at the two sides of the engine, is divided into a certain number of elements, i.e., ten elements for the heater and the water cooler, five hundred elements for the stack, one thousand elements for the resonator and the APA. Four parameters including the temperature and pressure amplitude in the left hot buffer, pressure amplitude in the reservoir of RC load and the operating frequency in the system are guessed and adjusted during the iterations to target the stack's cold end temperature of 300 K restricted by the water cooler, the pressure amplitude at APA's inlet equal to that at the tee connection of the engine, and the velocity amplitude of zero at the end of the right hot buffer. The distributions of pressure amplitude, velocity amplitude and temperature are finally obtained, as well as acoustic power generation and dissipation in stacks and tubes. The velocity mentioned in this paper is volumetric velocity.

Helium is adopted as working fluid in the simulation. Screen stack in the engine is treated with a capillary model. The local flow resistance induced by the variation of flow area is taken into account. In addition, the viscous resistances in the resonant tube and also in the APA, are corrected according to Ref. [14], since the oscillating flow is within the range of turbulence flow in the computed cases.

3. Simulation result and analysis

The distributions of pressure amplitude and velocity amplitude in the APA's of 8 mm diameter and various lengths are presented in Figs. 3 and 4, respectively. The location "0 m" stands for APA's outlet (the end connecting to the load, see Fig. 2). In the APA's of 4 m and 5 m in length, pressure amplitude increases monotonously from APA's inlet (the end connecting to the engine) to its outlet, accompanied by a monotonous decrease of velocity amplitude. However, for the length of 6 m and 7 m, a minimum of pressure amplitude (pressure node) and a maximum of velocity amplitude (velocity antinode) appear at almost the same location in the APA. The distribution characteristics, similar to a standing-wave acoustic field, are the reason for APA's amplification effect of pressure amplitude. Additionally, it is seen that a critical length for the occurrence of the abovementioned minimum and maximum lies between 5 m and 6 m. The different distributions have significant influence on the system's operating parameters, which will be discussed in the following sections.

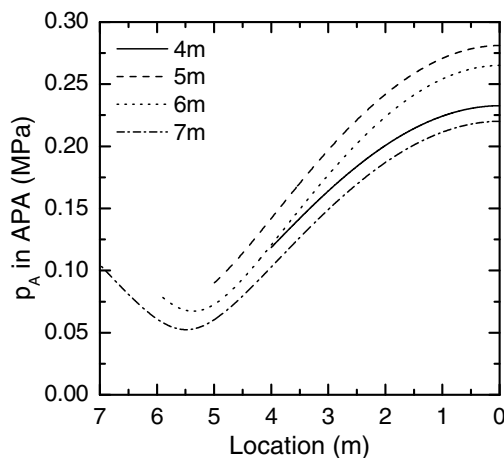


Fig. 3. Distribution of pressure amplitude in APA.

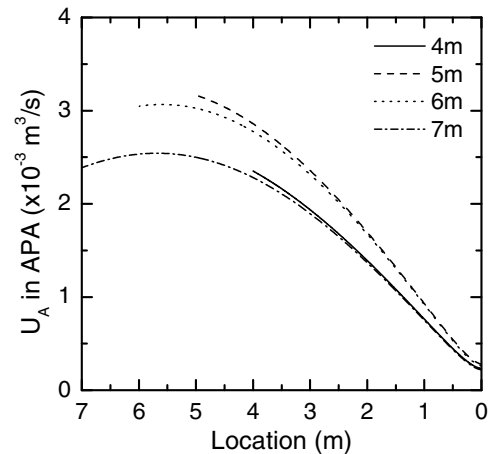


Fig. 4. Distribution of velocity amplitude in APA.

Variation of pressure ratios at APA's outlet and inlet versus its length with diverse diameters are shown in Figs. 5 and 6, respectively. For the pressure ratio at APA's outlet, a maximum appears at the abovementioned critical length, close to but less than a quarter of wavelength (roughly 5.8 m), except the case of 2 mm diameter. Contrarily, there is a minimum for the pressure ratio at APA's inlet. In the case below critical length, a monotonous distribution of pressure ratio lies in APA, as shown in Fig. 3, and the increase of length leads to its inlet approaching to the pressure node and also to the velocity antinode in terms of a standing-wave acoustic field, so the acoustic impedance at APA's inlet decreases markedly. The pressure ratio at APA's inlet is sustained by the engine. Although the pressure ratio at APA's inlet decreases with the increase of its length, there are two aspects helpful for a rise of pressure ratio at its outlet, both indicated in Figs. 3 and 4. One is that a longer APA means a longer distance for increase procedure of pressure ratio in APA, and the other is that a reduced acoustic impedance at APA's inlet leads to larger velocity amplitude in APA, and then to a higher increasing rate of pressure ratio (indicated by the curve's slope). However, situation becomes opposite when APA is longer than the critical length. A minimum pressure (pressure node) and a maximum velocity (velocity antinode) exist in APA. In this case, the increase of APA length makes its inlet deviate from the pressure node and also from the velocity antinode, so the acoustic impedance at the inlet rises. Although pressure ratio at the inlet increases with the prolongation of APA, the longer distance from the inlet to the pressure node results in a lower pressure ratio of the pressure node, and smaller velocity amplitude in APA, derived from the increased impedance at the inlet, leads to a lower increasing rate of pressure ratio in APA, as presented in Figs. 3 and 4. Thus, the further increase of APA length makes the pressure ratio drop at its outlet.

Fig. 7 presents the amplification ratio of APA, pressure amplitude at its outlet divided by that at its inlet, whose variation trend is similar to the pressure ratio at APA's outlet. The amplification ratio above 1 means an effective amplification effect. Curves in Fig. 7 show that the amplification effect can be realized with most dimensions of APA, except for a diameter of 2 mm.

Phase difference of pressure waves at APA's outlet and inlet is computed and shown in Fig. 8. The phase difference varies from 0° towards 180° with the increase of APA's length from 0 m to 8 m. A larger diameter leads to a sharper variation of the phase difference near the abovementioned critical length, which may be attributed to the weakened influence of acoustic resistance due to the increase of diameter. It is seen that characteristics of the

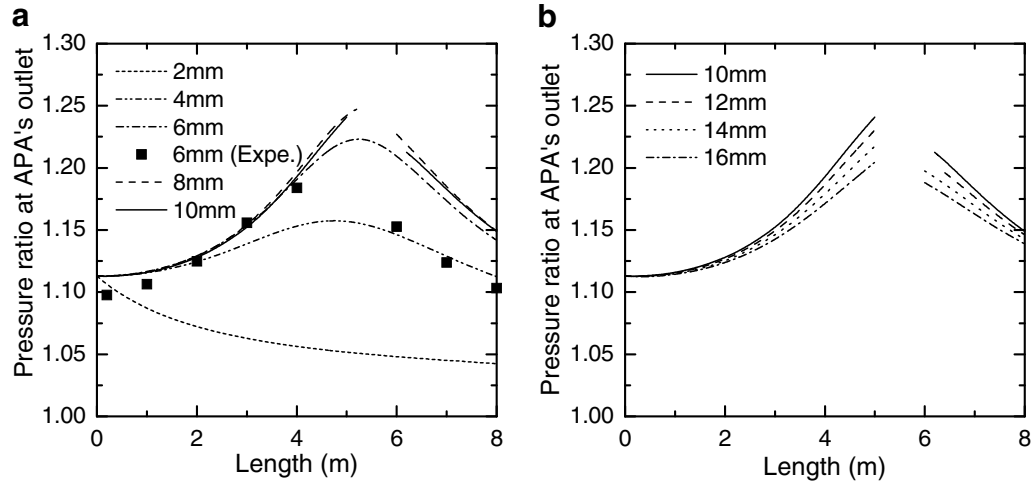


Fig. 5. Pressure ratio at APA's outlet versus dimensions of APA.

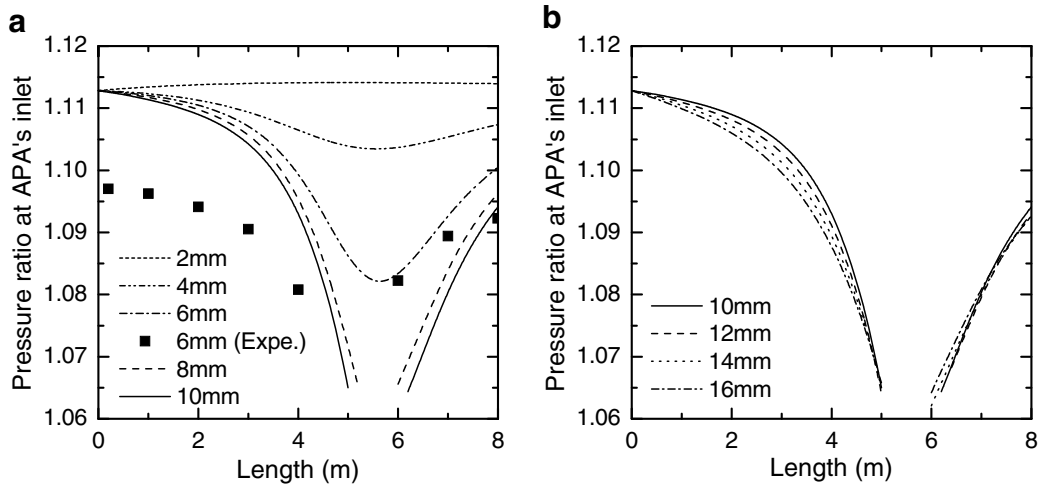


Fig. 6. Pressure ratio at APA's inlet versus dimensions of APA.

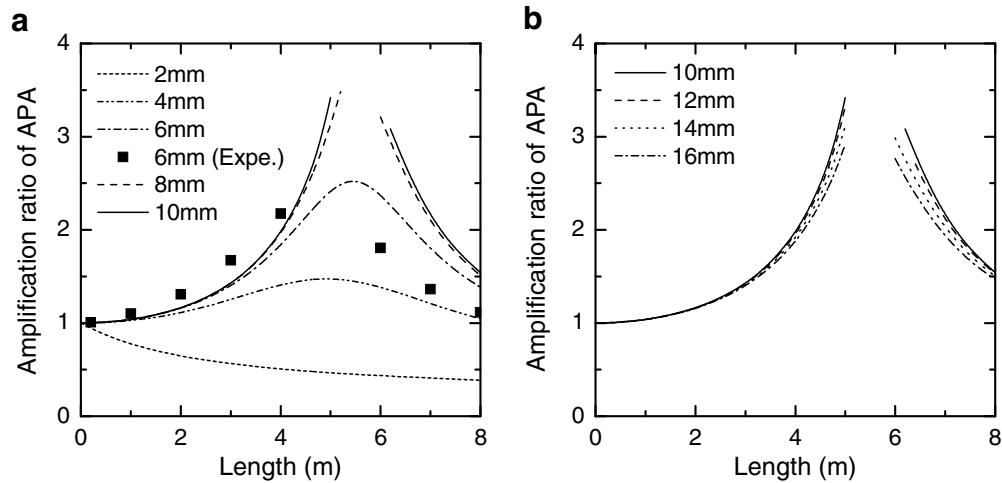


Fig. 7. Amplification ratio versus dimensions of APA, amplification ratio equals pressure amplitude at APA's outlet divided by pressure amplitude at APA's inlet.

phase difference are also similar to those of a standing-wave acoustic field.

An analysis of acoustic power generation and loss in the thermoacoustic system can discover more information about the influ-

ence of APA's dimensions on system operation. The variation of acoustic power generation in the stacks versus APA's length and diameter is shown in Fig. 9. For most dimensions, the introduction of APA leads to an obviously decrease of acoustic power generation

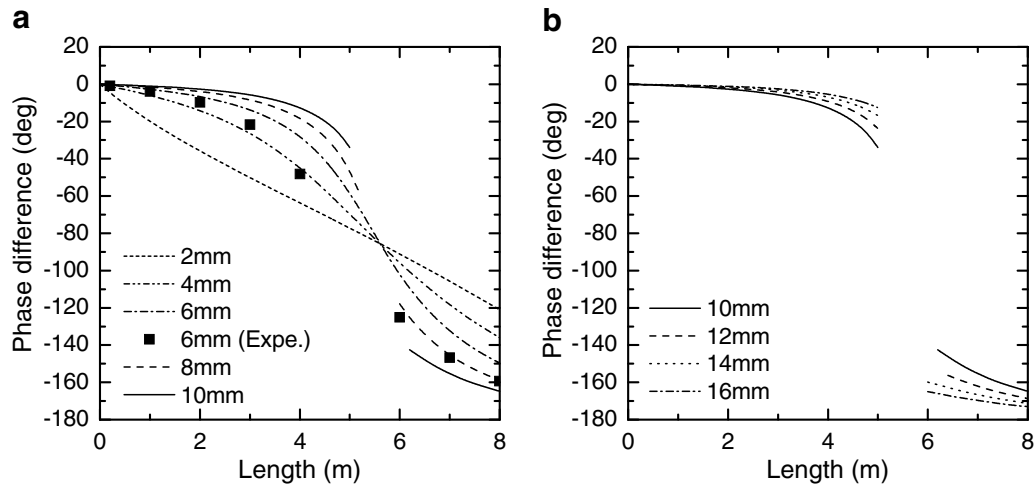


Fig. 8. Phase difference of pressure waves at APA's outlet and inlet versus dimensions of APA.

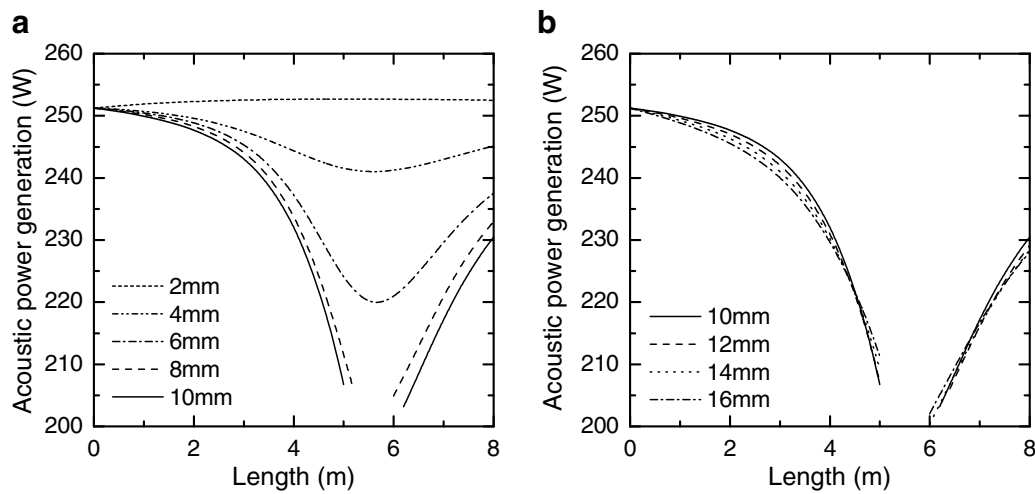


Fig. 9. Acoustic power generation in stacks versus dimensions of APA.

in the stacks. Acoustic power losses in the RC load, APA and engine are presented in Figs. 10–12, respectively. The amplified pressure ratio by APA at its outlet leads to an increased acoustic power delivered to the load. However, considerable acoustic power is

consumed by APA. So the acoustic oscillation is weakened by the introduction of APA, and the acoustic power loss in the engine is decreased. The forementioned drop of acoustic power generation is derived from the weakening of acoustic oscillation, especially

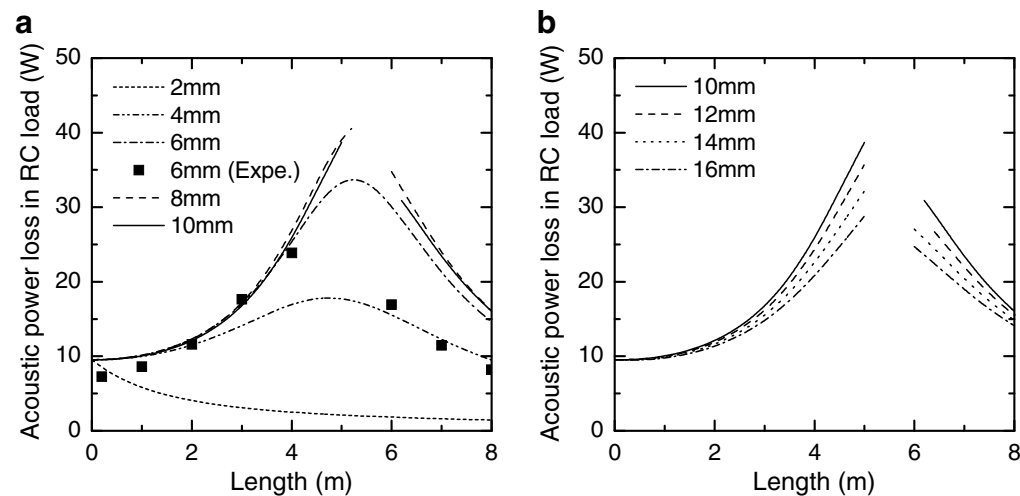


Fig. 10. Acoustic power loss in RC load versus dimensions of APA.

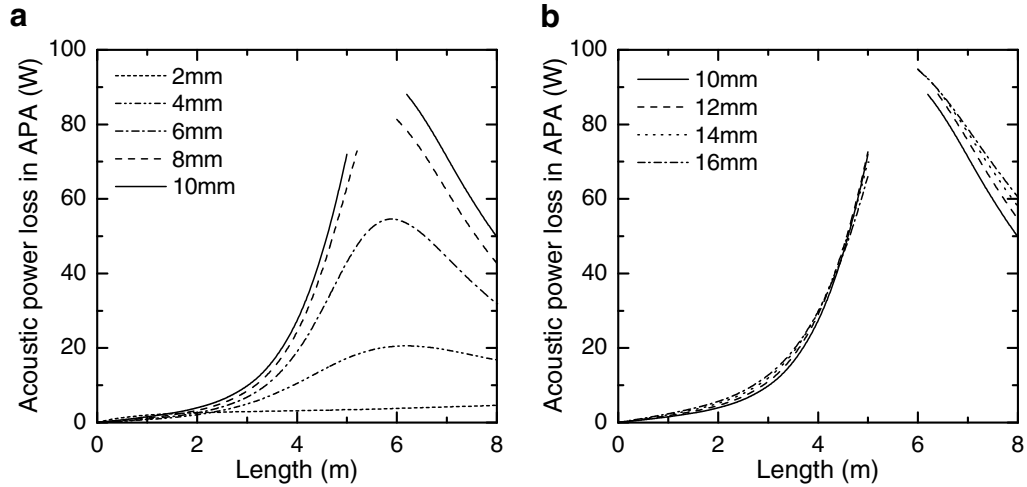


Fig. 11. Acoustic power loss in APA versus dimensions of APA.

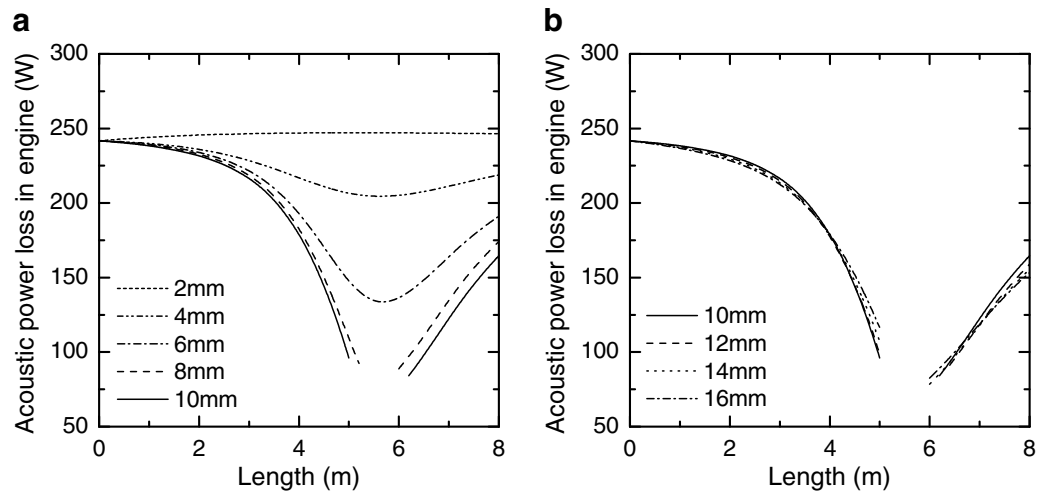


Fig. 12. Acoustic power loss in engine versus dimensions of APA.

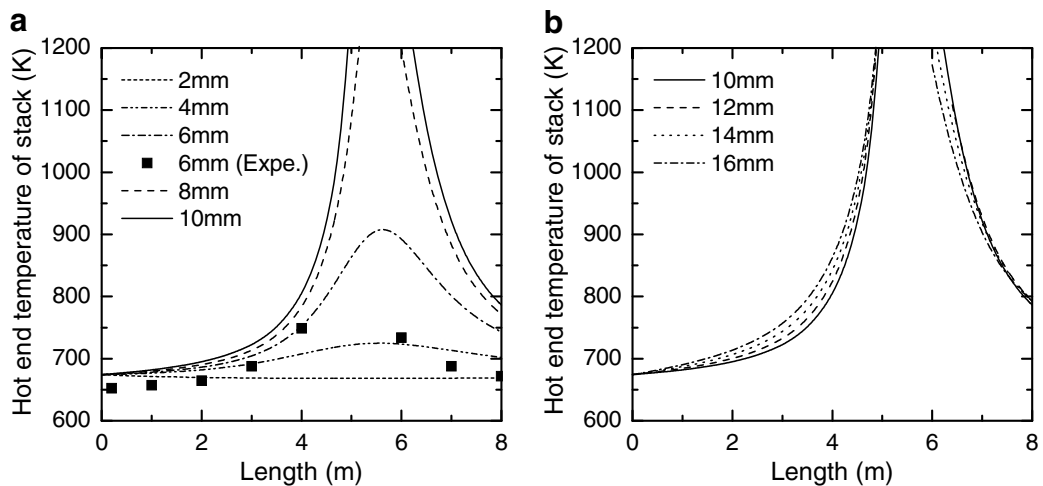


Fig. 13. Hot end temperature of left side stack versus dimensions of APA, similar for the right side stack.

the velocity amplitude, in the engine, because the acoustic power generation in stacks originates from the amplification of volumetric velocity [14]. In addition, the decrease of velocity amplitude in the engine weakens the heat transfer at the hot end of stacks, so

the amplified pressure ratio and the enlarged acoustic power delivered to the load are accompanied by a considerably high hot end temperature of stack, as shown in Fig. 13. Such a high temperature must be noticed in the practical application of APA. In the simula-

tion, data for the cases with temperatures above 1200 K are omitted, because it is too high for the brass screen stacks.

As mentioned before, the pressure ratio at APA's inlet is determined by oscillation status inside the engine, so the opposite variation trend of the pressure ratio at APA's inlet shown in Fig. 6, compared with that at APA's outlet presented in Fig. 5, also results from the weakening effect by APA to enhance the pressure ratio and acoustic power output towards the load.

The above discussion mainly focuses on the influence of APA's length, and this section comes to the diameter. For a diameter of 2 mm, significant acoustic resistance dominates the operation of APA, so the pressure ratio at its outlet and also the acoustic power delivered to the load monotonously drop with the increase of APA length, and no amplification effect can be achieved. Meanwhile, the significant acoustic resistance blocks the acoustic power entering the APA, thus, the change of APA length has little influence on the operating parameters of the engine, including pressure ratio at APA's inlet, acoustic power generation in stacks, hot end temperature of stacks, etc. The increased diameter can reduce the restraining of amplification effect by the acoustic resistance in APA, however, acoustic power loss in APA also rises. It is seen that, as a result, an optimal diameter for highest pressure ratio and acoustic power to the load is between 8 mm and 10 mm.

4. Experimental verification

In order to validate the simulation results, experiments with an APA of 6 mm diameter have been performed. According to the above simulation, helium with a mean working pressure of 2.6 MPa is adopted as working fluid and heating power input is 1.4 kW. The RC load consists of a needle valve and a reservoir of 155 cm³, as shown in Fig. 2, where the acoustic resistance of 1×10^9 is realized by adjusting the needle valve and the compliance impedance of the reservoir is roughly $-i \times 1 \times 10^8$ Pa s/m³ at the experimental conditions. Temperature and pressure measuring points are indicated in Fig. 2. The acoustic resistance and the acoustic power loss in the RC load can be estimated with the following equations:

$$R = \frac{p_2 - p_3}{U_{\text{load}}} = \frac{\gamma p_m |p_2|}{2\pi f V |p_3|} \sin \theta_{p_2-p_3} \quad (13)$$

$$\dot{W}_{\text{load}} = \frac{1}{2} \text{Re}[p_2 \tilde{U}_{\text{load}}] = \frac{2\pi f V}{2\gamma p_m} \text{Im}[p_2 \tilde{p}_3] = \frac{\pi f V}{\gamma p_m} |p_2| |p_3| \sin \theta_{p_2-p_3} \quad (14)$$

where V is reservoir volume, f is frequency, p_2 and p_3 are pressure amplitudes measured at P2 and P3 (see Fig. 2), and $\theta_{p_2-p_3}$ is phase difference between p_2 and p_3 .

Experimental data of pressure ratios at APA's outlet and inlet, amplification ratio of APA, phase difference of pressure waves at APA's outlet and inlet, acoustic power loss in RC load and hot end temperature of left stack have been measured and presented in Figs. 5–8, 10 and 13, respectively. The comparison of computed results and experimental data indicates that the simulation can reflect the variation trend of operating parameters with the change of APA's dimension. However, in the case of APA's length longer than 6 m, the computed values deviate from the experimental data. According to Fig. 4, a velocity antinode occurs in the APA, whose length is longer than 6 m. The computation error of acoustic resistance for the large velocity amplitude near velocity antinode may be the possible reason for abovementioned deviation.

In addition, there are no experimental data between the lengths of 4 m and 6 m in above figures. The APA's with the length of 4.5 m, 5 m and 5.5 m have been tested, but the acoustic oscillation in the system ceased for the APA's of 4.5 m and 5 m, and the acoustic oscillation varied from the fundamental-frequency mode to the second harmonic mode for 5.5 m APA, which roughly doubled the operating frequency and severely reduced the pressure ratio

in the system. The possible reason may be that the acoustic power dissipated in the whole thermoacoustic system is larger than the maximal acoustic power that could be generated in the stack at fundamental-frequency. The mechanism of the ceasing and the frequency transition needs to be further studied.

5. Conclusions

The simulation results indicate that the distribution characteristics of pressure and velocity in APA are similar to a standing-wave acoustic field, which is the reason for the amplification effect of both pressure ratio and acoustic power output. From the viewpoint of energy, the amplification effect results from the changed distribution of acoustic energy and acoustic power loss in the thermoacoustic system by APA. That is, APA elevates the pressure ratio and acoustic power output by weakening the acoustic oscillation in engine.

Analysis of the distribution characteristics is an effective method to study the impact of APA's dimensions on system's performance. A largest amplification effect can be achieved at the critical length for the occurrence of pressure node and velocity antinode in APA, close to but less than a quarter of wavelength. However, in this case, hot end temperature of stack is quite high, which may limit the application of APA for low grade heat energy.

Experiments have been performed to validate the simulation. The comparison of computed results and experimental data indicates that the simulation may roughly predict the variation of the operating parameters with the change of APA's dimension, when the system operates in the fundamental-frequency mode.

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